

## VIBRATION ANALYSIS AND FAULT IDENTIFICATIONS OF ROLLING ELEMENT BEARINGS - A REVIEW

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### ABSTRACT

*The rolling element bearings are the integral components of the rotor shaft system. The failure of bearing can lead to catastrophic consequences. To reduce the chances of failure in bearing, researchers have studied the dynamic and transient behavior of the mating bodies for the structural and vibration analysis. Various techniques were implemented and enough literature is available for the different conditions. In this review, various bearing rotor frameworks with different loading and fault condition were summarized. Moreover, different methods and techniques have increased the early prediction of the fault. Techniques like F. E. A proves to be promising tools to carry out the systematic dynamic behavior of the rotor-shaft system.*

**KEYWORDS:** F.E.A, Structural, Vibration, Rolling Element Bearing & Rotor-Shaft System

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### 1. INTRODUCTION

Rolling bearing elements are the essential and integral components of rotating machinery. Rolling ball bearings have been widely used in various industrial machinery. Since there is a variety of applications for the rotor-bearing system, it becomes a high degree of importance to study the transient behavior of rotatory machinery and bearings[1]. Subjected to heavy load applications, the bearing develops localized defects such as spalls, brinelling, dents, pits, and cracks. This makes the rolling contact fatigue life and vibration a by-product a major area of study [2]. The term 'vibration' is often studied under structural health monitoring (SHM) to evaluate life, reliability, productivity, and safety. The bearing health monitoring can easily uncover the performance and health of the ball bearing. A good deal of health monitoring techniques is being used like noise monitoring, motor current monitoring, temperature monitoring, vibration monitoring, etc. However, due to sensitivity to fault severity, vibration monitoring is being used to monitor the conditions of the bearings widely. Vibration analysis of the rolling element ball bearing is the function of their fault frequencies[3]. In recent years, new techniques for predicting bearing life and reliability have emerged. The new technique most of the researchers dealing with is 'simulation' to study the behavior of the fault frequencies. The simulation of faults using either MATLAB or any other F.E. A software like ANSYS, ADAMS, ABACUS, etc.

Researchers like **Purwo Kadarno and Zahari Taha**[4] have studied nodal force excitation on their dynamic model of the bearing by simulation. The various parameters like RMS and peak to the peak value of the

simulated signal and experimental signal were compared and satisfied. **Apandi et al.**[5] have proposed a study of the approach to study the frequency response of the faults using ANSYS and Concluded the presence of the non-synchronous peak in the frequency characteristics of the bearing signal.

## 2. DYNAMIC MODELING OF THE ROLLING ELEMENTS BEARINGS

The mass of the machine components, stiffness of the structure and external forces acting on the structure plays a vital role in any vibratory system. Therefore, many researchers have studied the relationships between bearing stiffness coefficient, damping coefficient, and dynamic force, etc. The contact stresses between the rolling element and the raceways are much higher than compared to the other stresses involved in the rotating machinery components. The Hertzian contact stress theory drives the relationship between the balls and the raceways over a contact area. The radius ( $a$ ) of the area of contact between the raceway and the rolling element is given by

$$a = \sqrt[3]{\frac{3F \left( \frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2} \right)}{4 \left( \frac{1}{R_1} + \frac{1}{R_2} \right)}} \quad (1)$$

Where,  $E_1$  and  $E_2$  represent the modulus of elasticity of spherical contacting bodies 1 and 2,  $\nu_1$  and  $\nu_2$  are the Poisson's ratios,  $R_1$  and  $R_2$  are the radii, respectively.

The given equation 2 represents the maximum contact pressure at the circular contact area is:

$$P_{\max} = \frac{3F}{2\pi a^2} \quad (2)$$

Where,  $F$  is the normal force.

The general and systematic study of the six-DOF dynamic model of a deep groove ball bearing is carried out. The effects of non-linear Hertzian contact deformations and elastohydrodynamic fluid film over the geometry imperfections, or so-called distributed defects and the localized defects are studied. The bearing forces and torsional components were calculated to propose the dynamic model by **Sopanen** [6]. The Newmark- $\beta$  and Newton-Raphson method were used as the solver for the 4-DOF dynamic model of the ball bearing considering the non-linear vibration effects. The results show good agreement between the parameters involved in the dynamic behavior of the bearing [7]. The angular contact ball bearing stability is the function of the friction threshold between the retainer and rolling elements, retainer instability, and frequency instability. These parameters were verified by the **Boesiger** [8]. Internal clearance plays a greater role in case of ball bearings the representative model of the rotor-bearing system was prepared by **Tomovic et al.** [9]. Further, the effects of internal radial clearance and the number of rolling elements were measured experimentally and analytically.

The vibration was the end result of the distributed defects and local defects. For the past decade, many new fault detection methods and techniques have been proposed by the researchers. In the next few sections, the models of a bearing system with local defects and distributed defects over the raceways were discussed. Moreover, the F.E. analysis of the rotor-bearing system was debated later.

### 2.1 Distributed Defect's Vibration Response of Rolling Element Bearing

The waviness present in the raceways, the surface roughness of the raceways, curvature of the rolling elements, imbalanced cage and misalignment are the distributed defects, which give rise to the additional vibrations in the rolling

bearing system which can excite the resonance in the system. The race waviness of the raceway can be expressed in the surface roundness profile of the bearing given by the formula in equation (2)

$$R(\theta) = \sum_{m=1}^n A_m \cos(m\theta + \phi_m) \quad (3)$$

where  $\phi$  = Phase angle of  $m^{\text{th}}$  order waviness,  $A_m$  = Amplitude,  $\theta$  = Angular coordinate with respect to the center of the bearing of the bearing ring.

The order and location of the waviness (inner/outer race) are two main factors that determine the frequency of waviness. Researchers in the past few decades have studied the influence of different order of waviness and irregular size ball bearing rolling element on the race waviness vibration response. The order of waviness generates vibration if near to the number of rolling elements ( $z \pm 1$  and  $z$ ) at the bearing fundamental fault frequencies [6]. Author of reference [10] has developed the 3-DOF bearing model and studied the effects of the outer race, inner race waviness and the off-size rolling element under radial load conditions. The order of waviness is equal to the number of rolling elements that give rise to spectral component peaks at inner race defect frequency. We concluded that the outer race spectral peak amplitude is much higher than the inner race spectral amplitude.

The analytical technique is proposed by Meyer et al. [11] while using Lagrangian time-displacement equations for the raceways of the bearing, with reference to the rotating ball forces to generate the model. The model is proposed for the vibration signature of the bearing under load due to the waviness and uneven ball curvature. Figures 1.1 (a) and (b) show the series of tones in the spectrum under uneven curvature of the rolling element and the misaligned outer, respectively.

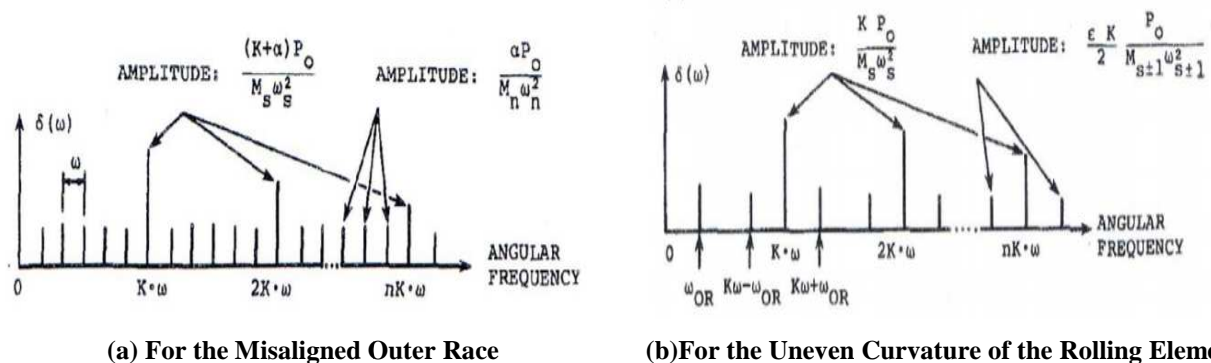


Figure 1.1: Frequency Spectrum [11]

Cao et al. [12] have extended the existing model of the 3-DOF deep groove ball bearing by considering the 5-DOF dynamic model of double-row self-aligning roller bearing (S.R.B). They extended the research by considering the impact of axial load/displacement in the direction of the axis. The condition for point contact and line contact were simulated under load conditions and parameters like surface waviness, radial clearance and defects were analyzed with force and displacement response of the Spherical roller bearing system. However, Babu et al. [13] concluded that the vibration response of the load-dependent frictional moment was high in amplitude compared to the unloading frictional moment, irrespective to the waviness order. And also proposed that, the inner race waviness is more responsible for the vibration generation when compared to the waviness of outer race and rolling elements. While the raceways were more of the concern, the integration with the rolling element, Ashtekar and Sadeghi [14] were developing a 3D Finite element model of the cage with respect to the exciting 6-DOF transient model of the ball bearing. The parameters influence like cage whirl,

shaft-misalignment, resulting ball motion were observed and noticeable effects like no sliding of the rolling elements in the presence of cage, fatigue failures were observed experimentally and in practice study.

Bearings are meant to run for a long time period under the different load conditions. The long-running time period of machinery produces the heat between the frictional contacting bodies, which give rise to the new study 'temperature' for the health monitoring and product life. This makes it essential to study the optimum temperature range for the bearing operation without failure. **Tarawneh et al. [15]** studied and developed the thermal finite element analysis for the railroad bearing for the optimum temperature range. The results for the heat generation in the elements of the bearing and surface temperature for bearing cup were discussed. **Randall et al. [16]** have proposed a model to deal with the background noise in the vibration signal, and to extract the deterministic and stochastic parts by the spectral correlations related to the cyclo-stationary machine fault vibration signals.

## 2.2 Localized Defect's Vibration Response of Rolling Element Bearing

The interaction between the local defects and the mating surfaces of the rolling element creates a greater amplitude contact stress and vibration pulses for the short duration of time. These short impulses can be observed **[17]** under the time domain and frequency domain analysis. These two different approaches were explained in the next sections.

### 2.2.1 Time Domain Data Analysis

The vibration signature of the bearing is generally evaluated by considering it in the time domain and frequency domain. In most of the time domain, the vibration analysis is summarised under the statistical variables like a skewness, crest factor, kurtosis and probability density curves, etc. In which, kurtosis is the greatest in degree recommended and efficacious parameter for the vibration signal analysis given in equation (3). The kurtosis response above 4 for the bearing defects and below 4 for the healthy bearing.

$$kurtosis = \frac{(N-1) \sum_{i=1}^N (x_i - \bar{x})^4}{\sum_{i=1}^N (x_i - \bar{x})^2} \quad (4)$$

Where,  $x_i$  = Expedient Amplitude,  $\bar{x}$  = Mean value,  $N$  = Rotational velocity in RPM.

The time domain study was done by **Paliwal [18]** in response to propose a new methodology for signal decomposition by spline wavelet functions and identify the various parameters like correlation coefficients, maximum mutual entropy. And **Karacay and Akturk [19]** experimentally determined the defects by using their characteristics fault vibration on the basis of crest factor, kurtosis, peak-to-peak amplitude, and RMS. But never explained the location of the defects.

The spectral kurtosis identification characteristics for fault were noted by **Dwyer [20]** and implemented by the **Antoni [21]** and for the first time, the concept of kurtograms was introduced. And the positions and a series of transients were identified in the frequency domain. The signal obtained from the bearing was coated with the noise, further to obtain the useful data. **Nikolaou [22]** has proposed a demodulation method to fully exploit the underlying feature of the vibration signal using Morlet wavelet family. The experimental studies of the method prove its validity. The shock pulse method **[23]** has the working principle of the transducer with the resonant frequency 32 kHz. The impacts by the defect in the bearings cause the shock impulse to start off the resonant frequency of the transducer by damped oscillations. The measure of the value, maximum to the corresponding damped transition gives a manifestation of the health of rolling bearings.

Electronic filters were used to eliminate the low-frequency vibrations produced by other than the rolling elements. The shock pulse generated by healthy bearings due to the order of waviness present and surface roughness of the raceways comes out to be the functions of the inner diameter of the bearing and shaft rotational speed. And hence the value called the initial value. To normalize the shock impact value, it is subtracted from the shock value of the test bearing. The maximum normalized value of the impact is the measure of the bearing operating condition. For the recent trends, **Srividya et al.** [24] have generated an automated detection methodology by extracting the time domain signal features for the fault analysis and proposed a neural network method for automated fault detection.

## 2.2.2 Frequency Domain Data Analysis

The fault signal impulses were generally the function of the fundamental fault frequencies and the rotating shaft speed in the frequency domain analysis. The fundamental fault frequencies highly depend on the geometric parameters and rotor speed, which can be calculated using the following equations (5 - 8) for the stationary outer race.

$$\text{Inner raceway ball pass frequency (BPFI)} = \frac{n}{2} \times F \times \left(1 + \frac{d_b}{d_p} \cos \theta\right) \quad (5)$$

$$\text{Outer raceway ball pass frequency (BPFO)} = \frac{n}{2} \times F \times \left(1 - \frac{d_b}{d_p} \cos \theta\right) \quad (6)$$

$$\text{Fundamental train frequency (FTF)} = \frac{F}{2} \times \left(1 - \frac{d_b}{d_p} \cos \theta\right) \quad (7)$$

$$\text{Ball spin frequency (BSF)} = \frac{d_p}{2d_b} \times F \times \left(1 - \left[\frac{d_b}{d_p} \cos \theta\right]^2\right) \quad (8)$$

Where  $n$  = number of rolling elements in the single raceway,  $d_b$  = ball element diameter,  $d_p$  = pitch diameter of the ball bearing,  $\theta$  = bearing contact angle,  $F$  = Shaft frequency.

In figure 2.1 (a) and (b), the sideband of the frequency response of the faulty inner race and outer race can be seen clearly. Whereas,  $f_s$  represent the shaft speed and the faults, and their frequency harmonics can be seen.

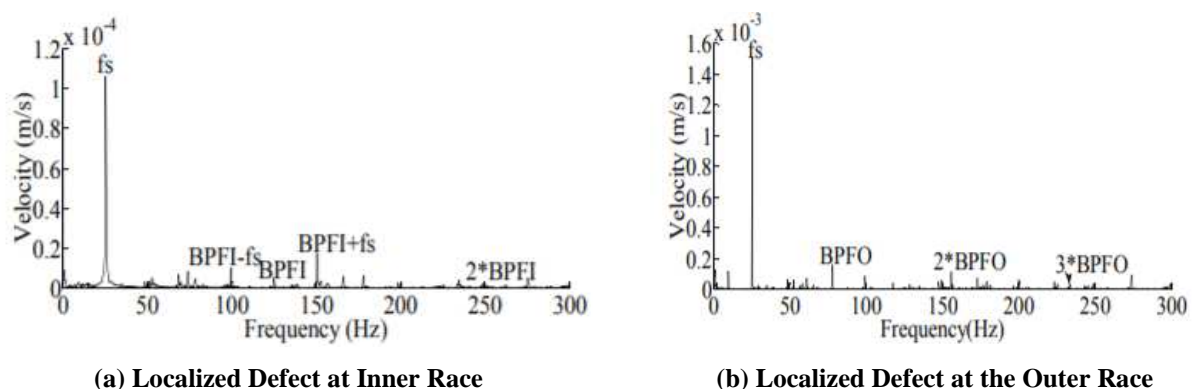


Figure 2.1: Frequency Domain Plots for Bearing having [25]

For the low-speed applications, the fault frequencies lie in the range of 500 Hz and below for the bearings. These are somewhat different from the calculated frequencies due to the consequences of skidding and slipping of rolling elements over the races [26]. Furthermore, the basic dynamic model was given by **McFadden and Smith** [27] for the multiple point defects at the inner race and faults were generated by the delta function. Accordingly, the radial load conditions generated by the Stribeck equation. The various parameters like non-linear and linear stiffness of the ball

bearings, the damping coefficients, tolerances, a slip of rolling elements, etc. were scoped to develop the generic model of the bearing system. **Patil [28]** have studied the analytical model of the deep-groove bearing with the varying sizes of the localized defects over different locations. The contacts were considered to be non-linear. Similarly, **Patel [29]** conducted research over the 6-DOF ball bearing model including the non-linear effects of the housing, mass of the shaft, races and rolling elements. And established that the vibration amplitude was waymore in case of multiple defects than the single defect.

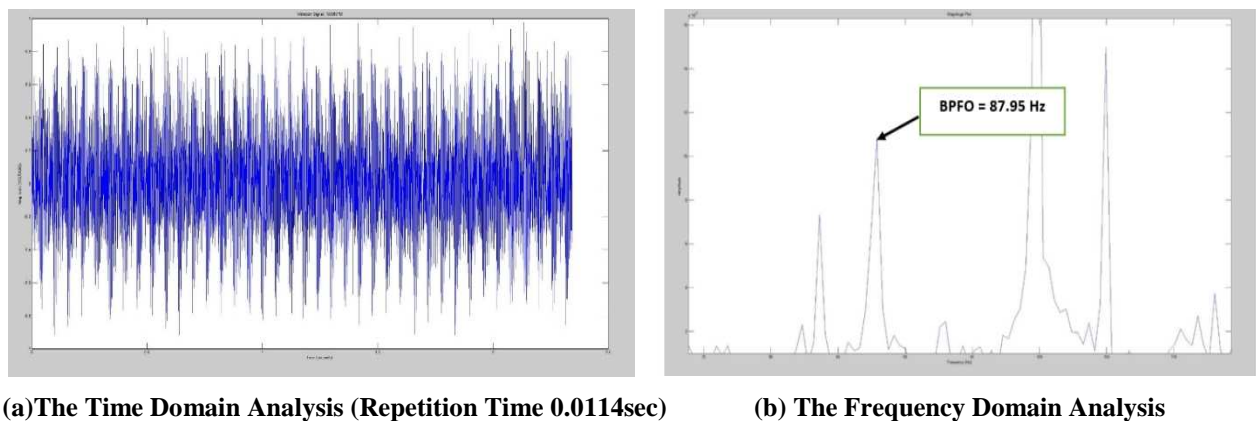


Figure 3.1: The Gaussian Impulse Train

Researchers have explained the high energy impact generated by the rolling elements when interacted with the fault for a very short time period. These impacts can galvanize the natural frequency of the rotor-bearing structure. The appearing and disappearing of the peaks in the spectrum were well explained by **James [30]**. Moreover, **Tandon and Nakra [31]** have predicted the natural resonance frequencies to be more than 5 kHz. And can be calculated by the given set of equations(5-8). For the fault feature analysis, **Martin and Thorpe [32]**proposed a signal enveloping technique by normalizing the signal in accordance with the healthy bearing which increases the responsetowards fault detection. But due to its limitations, it was unable to detect the advance faults with the background noise. Researchers like **Burgess [33]** argues that the increase in the background noise to the amplitude of the fault signal was due to the leading edge of the fault mixed with the decay of the previous impact as defect progresses.

Signal enveloping and the spectral analysis are the most reliable approaches were proposed by the researchers. In accordancewith that, **Kankar [34]**defined a new technique based on the wavelet decomposition using (M. S.E.C) Minimum Shannon Entropy Criterion. Seven level wavelets were considered, extracting the features of the raw signal. Further, the study was extended[35] to surface the underlying features of faultsof the bearing by the response surface methodology. **Mohammadi et al. [36]**have proposed the high-frequency resonance method, which envelopes the signal fault and multiple defectssignals in the frequency domain, and the amplitude and pattern of the signal were compared to the simulated modelfault response curve to check the accuracy of the proposed model of the bearing with faults and healthy bearing.

**Weinzapfel et al. [37]** have extensively studied the 3D finite element topology of the grains microstructure for the spalling in CRB influencing the contact fatigue in roller bearings. Extended independent work of **Raje et al [38]** has considered the geometric imperfections and uneven distribution of the material property for thepropagation of sub-surface spalling in the developed fatigue model. Correlation of the spalling life to the contact pressure with the stress-life coefficient 9.35 followed the inverse power law.



### 3. EVOLUTION OF THE BEARING FAULT STUDY

For many years, the vibration and the problems connected to it in the bearings have created a general field of research in the research society. The complexity of the information obtained is concerned about a certain type of fault and parameters like fatigue, waviness, defects, etc. Various techniques have been developed over the past few decades to obtain this information from the bearing with the aid of signal processing like time and frequency data analysis, high resonating frequency analysis, etc. Various other approaches like signal enveloping, adaptive noise cancellations, and neural network approach were identified and implemented to extract the useful information from the raw signal.

The enveloped signal features less importance when comes to detect the incipient defects. To rectify this problem, the method of spectral kurtosis, center frequency and band width selection was proposed and studied by the **Patel [39]**. Moreover, the Hilbert-Huang transformation technique was proposed by the **Li and Wang [40]** due to its high-resolution analysis for the dynamic vibration signals. Somewhere, signal analysis methods were proposed using discrete wavelet decomposition, Laplace transformation, etc. **[41-42]**. **Prabhakar [41]** have proposed a more accurate model by using discrete wavelet decomposition. **Mohanty et al [42]** concluded that the low power frequency signal was suppressed under the noise in the signal and developed a variational mode decomposition method, followed by the FFT to amplify features of the faults for the non-stationary signals.

Recently, a new class of ball bearing analysis has been emerged considering the behavior of the bearing forces, contact stresses and characteristics fault frequencies. By ‘Simulation by F. E.A/C.A.E’, finite element based software was used to develop the real-time condition over the developed 3-D CAD model of the rotor-bearing system to generate the generic results for the forces involved, acceleration results, stresses, etc. Therefore, **Nabhan [43]** have discussed the load distribution effects on the outer race housing due to defects and simulate it with the ABACUS/CAE software. And, **Zhaoping [44]** constructed a 3-D model in the ANSYS APDL software to simulate the effects like frictional stresses, penetration, deformations and contact stress between the outer race, inner race, and cage, rolling elements. Whereas, **Yang et al. [45]** have considered the three different models of the bearing and created 3-D finite element models to study the dynamics of the faults behavior in the ADAMS software. However, **Xintao et al. [46]** have considered the crowing value of the tapered roller bearing to simulate the dynamic 3-D model in ANSYS LS-DYNA. The bootstrap method was used to illustrate the vibrational acceleration data. Furthermore, **Tyagi and Panigrahi [47]** have concluded that it is difficult to obtain a vibration signature from the bearing fault at its initial stage, so it is important to develop a CAD model to represent the real-time conditions and obtain the transient analysis results. The nodal forces were applied over a 3-D finite structure of the bearing to simulate the actual force conditions. The vibration signature was noted with the increase in the directional acceleration and nodal forces.

### 4. CONCLUSIONS

The extensive study of literature has proved that fault identification in the bearing can help to reduce the chance of failure and can increase the life, reliability, etc. of bearing. The various identification methods like time-domain analysis and frequency-domain analysis can increase the sensitivity toward the fault early prediction. The techniques of signal enveloping like, FFT, Laplace transformation, Hilbert-Huang transformations, etc. have adapted to study the vibration signature in high resolution. For recent analysis,

the F. E. A, ANSYS, and ADAMS were proved to be areliable tool for the structural and dynamic study of the bearing framework.

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